Accurate Distributed Inductance of Spiral Resonators

Z. M. Hejazi, *Student Member, IEEE*, P. S. Excell, *Senior Member, IEEE*, and Z. Jiang

***Abstract*—A simple procedure for calculation of the distributed inductance of spiral resonators and micro-inductors is derived, including all possible mutual inductances. It is applicable for spiral configurations from rectangular to smooth circular. Using this procedure and an accurate formula for distributed capacitance of a spiral resonator, the resonant frequency was accurately predicted. If the surface resistance of the conductors is known, the unloaded quality factor of any spiral resonator can also be calculated.**

***Index Terms*—Computer simulation, filters, planar inductors, spiral resonators, superconductors.**

I. Introduction

The knowledge of distributed inductance and capacitance enables more accurate prediction of resonant frequency and other parameters of spiral structures used as resonators and inductors in planar circuits. A method for prediction of distributed capacitance of spiral resonators was reported in [1] and tested in prediction of the resonant frequency F_o of spiral resonators, in combination with a standard inductance formula. Applying expressions available in the literature to given spiral dimensions, the disagreement in the inductance values varies from 30% to 100%. None of the methods used was derived for circular thin film spirals with the mutual inductances taken into account. Based on Grover [2], Greenhouse [3] expanded a method, which included the mutual inductances of all arms of a square spiral inductor. Gopinath *et al.* [4], [5] calculated the inductance, and its variation with frequency, of finite-length strips using a moment method. Djordjević *et al.* [6] also used a method of moments similar in theory to [5] but for a much wider class of geometries: It was applied to rectangular spirals. Schmückle used the method of lines [7], including negative mutual inductance, but still restricted it to rectangular spirals. Although these numerical techniques are claimed to be accurate, they are limited to certain geometries. Applying them to smooth circular spirals, if possible, would require large computational resources.

II. Method of Analysis

In the present work, the spiral was divided into segments with a resolution from $\pi/180$ (finest circular) to $\pi/2$ (square spiral) with combinations of pairs of segments producing mutual intersegment inductances. The pair combinations include four types: 1) segments meeting at a point with known bend angle ($\pi/2 \leq \psi < \pi$) [see Fig. 1(a) and (b)]; 2) segments not meeting at a point but their extensions subtend an angle ($0 < \xi < \pi$) as shown in Fig. 1(c); 3) parallel straight segments on the same side of the spiral; and 4) parallel segments on opposite sides of the spiral [see Fig. 1(d) and (e)]. All these configurations are building blocks of any spiral shape such as rectangular, hexagonal, octagonal, through to smooth circular [see Fig. 2(a)–(d)]. Calculation of the self inductances of the individual segments and the mutual inductances of all possible combinations in any spiral shape would directly yield an exact distributed total inductance of the structure. This method will also enable the calculation of the inductance of complex configurations such as logarithmic spirals (where the conductor width and/or spacing are variable).

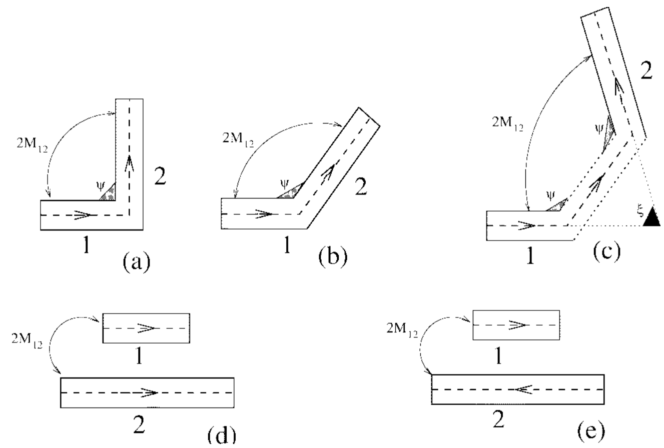

Fig. 1. Segment pair combinations contributing to any spiral configuration and their common equivalent circuit. The arrows indicate the current direction and θ is the segment resolution.

The different mutual field interactions between the segments is controlled by their mutual angles, length, width, geometric mean distance (G_{md}), and the current direction indicated with arrows in Figs. 1 and 2. The general equivalent circuit of an arbitrary bend is shown in Fig. 1(f). Fig. 1(g) shows the mutual inductances M_{12} and M_{21} that result from two generated currents i_1 and i_2 with the same frequencies. M_{12} is the mutual inductance of L_1 and L_2 caused by current i_1, while M_{21} is caused by i_2. In the case of acute angles, the mutually induced electromagnetic fields (EMF's) would aid the self-induced EMF's and in the obtuse angle case they would oppose them. In the case of parallel segments [Fig. 1(d) and (e)] when

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Z. M. Hejazi and P. S. Excell are with the Department of Electronic and Electrical Engineering, University of Bradford, Bradford BD7 1DP, U.K. (e-mail: Z.Hejazi@bradford.ac.uk; P.S.Excell@bradford.ac.uk).
Z. Jiang is with the Institute of Electronics, Academia Sinica, Beijing 100080, China.
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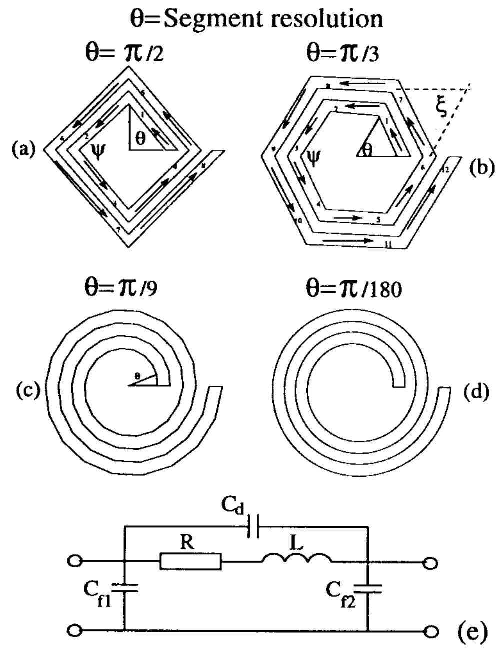

Fig. 2. Segment resolution transition from a square to a smooth circular spiral and its equivalent circuit.

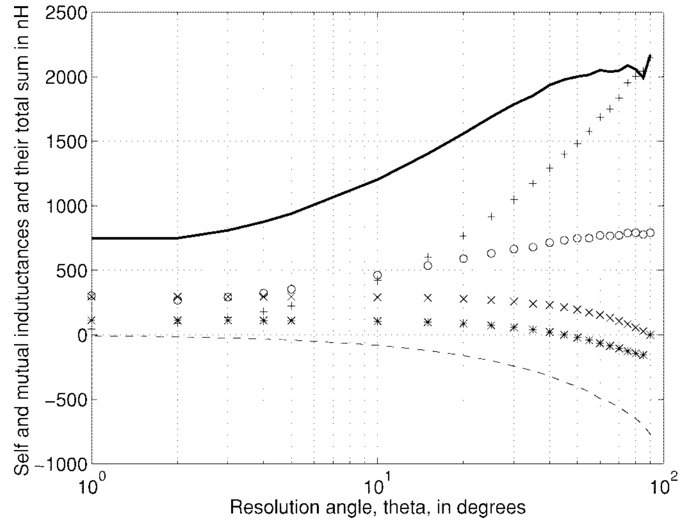

Fig. 3. All inductance terms and the total sum L_T against resolution angle θ for a fixed number of turns $N = 10$. $\times\times\times$: Total M of segments meeting at a point, ***: Total M of segments not meeting at a point, ooo: L_o, +++: Total M^+, $---$: Total M^-; solid line: L_T of the spiral.

the current direction in both segments is identical, their mutual inductance is to be added and referred to as positive M^+. When the current flows in opposite directions, the mutual inductance is to be subtracted and referred to as negative M^-, where M^+ or M^- equals $2M_{12}$ for this combination type. The mutual inductances between adjacent segments meeting at a point $M_{j,j+1}$ (as a function of $\cos\psi$) and the nearest segments not meeting at a point ($M_{k,k+2}$), as a function of $\cos(\xi)$, are also taken into account, where $j = 1, 2, 3, ...n - 1$ and $k = 1, 2, 3, ...n - 2$, and n is the maximum number of segments in the spiral. Applying the summation method, the total distributed inductance of any spiral shape would have the general form

$$L_T = \sum_{i=1}^{n} L_{o_i} + 2 \cdot \left[\sum M^+ - \sum M^- + \sum_{j=1}^{j} M_{j,j+1} \pm \sum_{k=1}^{k} M_{k,k+2} \right] \quad (1)$$

where L_{o_i} are the self-inductances of the segments. The fourth term is always positive because ψ in any spiral is never acute (except in a triangular spiral). The last term is positive when $\xi < 90^o$ [see Fig. 1(h)] and equals zero when $\xi = 90^o$. It changes sign when $\xi > 90^o$. Obviously, these last two terms should not exist in a rectangular spiral. For a one-turn spiral, the second term vanishes. By this way, an accurate total inductance may be calculated.

III. Computational Results and Comparisons

All computations were carried out with the same spiral dimensions: $S = W = 1.5$ mm, $D_{in} = 4$ mm, dielectric height $h = 0.62$ mm, $\varepsilon_r = 9.5$. S is the spacing between turns, W is the conductor width, and D_{in} is the inner diameter of the spiral. Fig. 3 shows all inductance terms and their total sum against resolution angle θ for a fixed number of turns $N = 10$. The examples available for comparison with two alternative methods [7] are square spirals. The spiral inductance is compared with these models in Table I, where $N = 2$, $h = 318\,\mu$m, $W = 50\,\mu$m, and $\varepsilon_r = 9.8$. Note that S_i is the inner side and S_o is the outer side of the square spirals. Fig. 4 shows a comparison between the present and previous methods [2], [3], [8], [9] for a given square spiral. However, these previous methods are restricted to square spirals while the present method is not. The computation time required by the present method depends on θ and N. On a Sun sparc20 computer, it was a fraction of a second for the examples given in Table I, while in the examples given in Figs. 3 and 4, it varied from 1 to 5 s.

TABLE I
Comparison with Fig. 8 of Schmückle's Example Spirals [7]
Where $N = 2$, $h = 318\,\mu$m, $\varepsilon_r = 9.8$ $h = 318\,\mu$m, and $\varepsilon_r = 9.8$

S and W μm	Side dimensions μm	A[7] nH	B[7] nH	This Work nH
$S = 0.6W$	$S_i = 318, S_o = 478$	2.7	2.2	2.57
	$S_i = 635, S_o = 895$	5.8	5.25	5.89
$S = W$	$S_i = 318, S_o = 618$	2.8	2.2	2.6
	$S_i = 635, S_o = 935$	5.2	5.8	5.68
$S = 2W$	$S_i = 318, S_o = 718$	3.2	2.7	3.14
	$S_i = 635, S_o = 1035$	5.9	5.3	6.0

IV. Prediction Accuracy for Lossless/HTS Spiral Resonators

Using thin-film spiral resonators made of YBCO and TBCCO high-temperature superconducting materials (HTS) fabricated and tested [10] for mobile telephone bands, the measured and calculated F_o were compared. The spiral dimensions were $S = W = 0.5$ mm, $D_{in} = 2.8$ mm, $D_{out} = 8.8$ mm, and film thickness $t_s > 3\,\mu$m. The dielectric was $LaAlO_3$ with thickness $h = 0.508$ mm and $\varepsilon_r = 24$. The distributed capacitance C_d [see the equivalent circuit

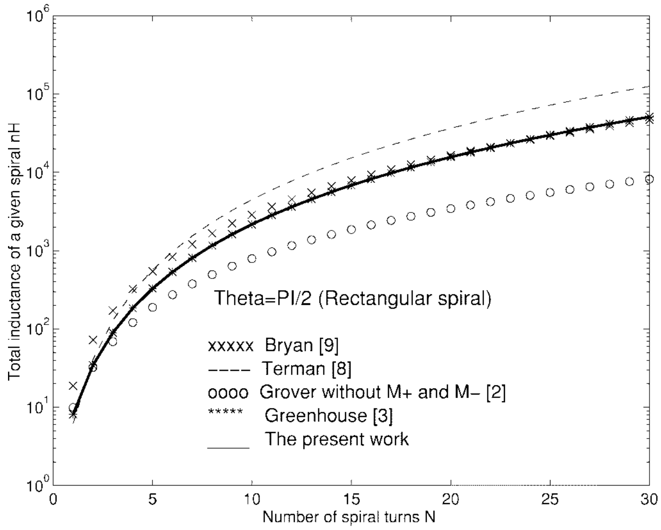

Fig. 4. Comparison of square spiral inductance calculated with different methods.

in Fig. 2(e)] calculated without the end-effect (fringing) capacitances C_{f1} and C_{f2} [1] was found to be $C_d = 0.554\,55$ pF ($C_f = C_{f1} + C_{f2} = 0.4252$ pF). The total capacitance C_T of the HTS spiral is then $C_T = 0.979\,76$ pF. The total distributed inductance L_T calculated in this work is $L_T = 35.6$ nH. F_o calculated by the well-known formula for loss-free conductors $F_o = 1/(2\pi\sqrt{(L_T C_T)})$ is $F_o = 852$ MHz. The measured F_o of the YBCO and TBCCO resonators was 840 and 844 MHz, respectively, which shows good agreement. Using L_T and the surface resistance of the materials (280 and 160 $\mu\Omega$ for YBCO and TBCCO, respectively) with film thickness of 10 μm, the unloaded $Q-$factor, Q_o, for this structure is calculated as 6075 and 10 632 for YBCO and TBCCO, respectively. Q_o for an identical copper resonator is 223.

The capacitance values strongly depend on the dielectric parameters while the inductance does not. For instance, the same structure realized in copper on an alumina substrate with $\varepsilon_r = 9.5$ and $h = 0.62$ mm yields $C_f = 0.3761$ pF and $C_d = 0.402$ pF, i.e., $C_T = 0.7786$ pF and $F_o = 955.86$ MHz. On a thin Duroid substrate with $h = 0.127$ and $\varepsilon_r = 2.2$, $C_f = 0.189$ pF and $C_d = 0.2094$ pF, i.e., $C_T = 0.398$ pF and $F_o = 1.3364$ GHz.

V. Conclusion

This method is simple and suitable for a CAD routine with minimum computation time and cost and gives results agreeing with numerical methods using a method of moments and the method of lines. It has no restriction to rectangular or Archimedean spirals. Using accurate distributed inductance and capacitance (with open-end effect), F_o of HTS spiral resonators were accurately predicted. Using this inductance and the surface resistance of the conventional or HTS conductor, the unloaded quality factor of any spiral resonator can also be calculated.

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